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Geometric Hierarchy and the Invisible Axion

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ABSTRACT

We propose a class of spontaneously broken locally supersymmetric grand unified theories, where both, the weak doublet and the color triplet of Higgs are massless in the supersymmetric limit. Due to supersymmetry breaking, the Higgs triplet acquires a mass of order $\sqrt{mM}$ ($\sim 10^{10}$GeV) at the one loop level, m and M being the gravitino mass and the grand unification mass respectively. The Higgs doublet acquires a vacuum expectation value (vev) of order $\sqrt{m\sqrt{mM}}$ ($\sim 10^{6.5}$GeV). Starting from a higher grand unification gauge group e.g. SU(6), we can push down the $SU(3)^c \times SU(2)^L \times U(1)$ breaking scale to the same order of magnitude as the gravitino mass m ($\sim 10^3$GeV). These models naturally admit invisible axions with decay constant of order $\sqrt{mM}$ ($\sim 10^{10}$GeV).

I. INTRODUCTION

Supersymmetric grand unified theories[1] are of interest at present, since they provide a partial solution of the hierarchy problem[2]. In the exactly supersymmetric limit, once a large mass hierarchy is established at the tree level, it remains stable under radiative corrections. But since supersymmetry is not a good symmetry of nature, it must be either softly or spontaneously broken in such a way that the large mass ratios still remain stable under radiative corrections. A more recent approach to the problem[3] starts with a locally supersymmetric grand unified theory, with supersymmetry spontaneously broken by the super Higgs mechanism. The super Higgs sector is coupled to the observable sector (which involves all the usual fields in the SU(5) GUT) only through the effect of gravity. In such a case, the effective Lagrangian in the observable sector may be written as a sum of the exactly supersymmetric SU(5) Lagrangian and a set of soft supersymmetry breaking terms, whose mass scale is set by the gravitino mass.

But even if supersymmetrv can protect the mass hierarchy from radiative corrections, there remains a second hierarchy problem, why is the colored Higgs triplet so heavy compared to the SU(2) doublet? Two different mechanisms have been proposed to answer this question, the sliding singlet mechanism[4] and the missing partner mechanism[5]. Of these, the sliding singlet model has been shown to be unstable under soft supersymmetry breaking[6]. Under one loop radiative corrections the Higgs doublet acquires a mass of order $\sqrt{mM}$, where m and M are respectively the gravitino mass and the grand unification mass. The missing partner model is free from this problem, but it requires a large number of Higgs.

It has also been pointed out that in supersymmetric GUTs, in order to get the correct baryon to photon ratio in the present universe, the triplet Higgs mass has to be of order 10^{10}GeV, which is considerably lower than the grand unification mass. We may call this the third hierarchy problem. In this paper we propose a class of models in which both the doublet and the triplet of Higgs are massless in the exactly supersymmetric limit. When we introduce the soft supersymmetry breaking term, the triplet Higgs acquires a mass of order $\sqrt{mM}$, whereas the doublet Higgs acquires a vacuum expectation value of order $\sqrt{m\sqrt{mM}}$, due to radiative corrections. Taking $M \sim 10^{17}$GeV, and $m \sim 10^{3}$GeV, we get a triplet mass of order 10^{10}GeV, and a doublet Higgs vev of order $10^{6.5}$GeV. On the other hand, if we start from an SU(6) gauge theory and assume that it breaks down to $SU(3) \times SU(3) \times U(1)$ at the GUT scale M, then it is possible tp arrange that the residual symmetry breaks down to $SU(3) \times SU(2) \times U(1)$ at a scale of order $\sqrt{m\sqrt{mM}}$ and the

$SU(3)\times SU(2)\times U(1)$ symmetry is broken to $SU(3)\times U(1)$ at a scale of order m. These models contain $SU(3)\times SU(2)\times U(1)$ singlet fields with vev$\sim 10^{10}$GeV and hence one can introduce invisible axions in these models, with decay constant $\sim 10^{10}$GeV, without the need of fine tuning any parameter.

Sec.II of the paper gives an example of the class of models where the doublet and the triplet masses are generated by the one loop radiative corrections. In Sec.III we explain how to introduce invisible axions in our model. The axion decay constant naturally comes out to be of order 10^{10}GeV. In Sec.IV, we propose a model, based on the SU(6) gauge group, in which the $SU(3)\times SU(2)\times U(1)$ breaking scale may be kept as low as the gravitino mass. We summarize our results in Sec.V.

II THE MODEL

Our model consists of a set of heavy superfields Φ, R, $\tilde{R}$, which, for definiteness, will be taken to be in the 24, 10 and $\overline{10}$ representations of SU(5). We also have a set of light superfields $\Sigma(24)$, $S(1)$, $H^{(i)}(5)$ and $\tilde{H}^{(i)}(\bar{5})$, where i runs over the number of Higgs multiplets we want in the theory.

The superpotential is,

$$W = \frac{1}{2} M_1 \Phi^2 + \frac{1}{3}\lambda_1 \Phi^3 + M_2 R\tilde{R} + \lambda_2 \Phi \tilde{R} R + \frac{1}{3}\lambda_3 \Sigma^3$$

$$+ \sum_{i=1}^{n} \lambda_4^{(i)} \Sigma \tilde{H}^{(i)} H^{(i)} + \sum_{i=1}^{n} \lambda_5^{(i)} S \tilde{H}^{(i)} H^{(i)} + \lambda_6 \Sigma R\tilde{R} \qquad (1)$$

where the mass parameters M_i are of the order of grand unification mass. For simplicity, we have dropped all the group indices, and also the quark-Higgs interaction terms. In the supersymmetric limit, Φ acquires a vev of order M_1/λ_1, which breaks SU(5) to SU(3)×SU(2)×U(1). There are, of course, other degenerate vacua which are SU(5) symmetric and SU(4)×U(1) symmetric respectively, but we do not consider them here. All the other fields have zero vev. The fields Σ, S, $H^{(i)}$ and $\tilde{H}^{(i)}$ remain massless in this limit.

The effect of supersymmetry breaking by super-Higgs mechanism is to introduce soft supersymmetry breaking terms in the action of the form[3],

$$\int d^2\theta\ \eta \left(\sum_i y_i \frac{\partial W}{\partial y_i} + (A-3)\, W(y) \right) + \sum_i \int d^2\theta\, d^2\bar{\theta}\ \bar{\eta}\, \eta\, \bar{y}_i\, y_i + \text{h.c.} \tag{2}$$

$$\eta = m\, \theta^2 \tag{3}$$

m being the gravitino mass. y_i's denote the set of all superfields Φ, Σ, R, $\tilde{R}$, S, $H^{(i)}$ and $\tilde{H}^{(i)}$. Due to the presence of these explicit supersymmetry breaking terms, there will be higher loop radiative corrections to the effective action of the form[8]

$$\int d^4\theta\ f(\eta, y_i, \bar{\eta}, \bar{y}_i) \tag{4}$$

where f is a polynomial in η, $\bar{\eta}$, y, $\bar{y}$ and their covariant derivatives. The terms responsible for producing large masses or vev's of the light fields have the following form,

$$m F_\Sigma f_1 (\Phi, M) + m F_S f_2 (\Sigma, S) + m^2 \Sigma f_3 (\Phi, M) + m^2 S f_4 (\Sigma, S) \tag{5}$$

In the above equation, Σ, S, Φ denote the first components of the corresponding superfields, whereas F_Σ, F_S denote the auxiliary components of the corresponding superfields. f_i's are functions of there arguments. Typical diagrams contributing to f_1 and f_2 in one loop order have been shown in Fig.1. For simplicity, we shall drop the functions f_3 and f_4 in our future discussion, since they do not qualitatively change any of the discussions that will follow. Typically, $f_1 \sim M, \Phi$; $f_2 \sim \langle\Sigma\rangle, \langle S\rangle$. We have ignored the

Σ or S dependence of f_1, since, as we shall see, $\langle\Sigma\rangle$ or $\langle S\rangle$ are small compared to $\langle\Phi\rangle$ or M. We shall carry out the analysis including terms of the form $mF_\Sigma f_1 + mF_S f_2$ in our Lagrangian, but throwing away all other terms which may arise from (4). The important point to note is that there is no coupling of F_S to the grand unification scale M. The first $F_S M$ coupling comes at the three loop level, which can be kept sufficiently small ($\sim F_S f_2$).

Adding (5) and (2) to the Lagrangian given by the superpotential W in (1), and eliminating the F components of various fields through their equations of motion, we get the effective potential,

$$
\begin{aligned}
V = & \tfrac{1}{2}\left| M_1 \Phi + \lambda_1 (\Phi^2)_{24} + \lambda_2 (R\tilde{R})_{24}\right|^2 + \tfrac{1}{2}\left| M_2 R + \lambda_2 \Phi R + \lambda_6 \Sigma R\right|^2 \\
& + \tfrac{1}{2}\left| M_2 \tilde{R} + \lambda_2 \tilde{R}\Phi + \lambda_6 \tilde{R}\Sigma\right|^2 + \tfrac{1}{2}\Big| \lambda_3 (\Sigma^2)_{24} + \sum_i \lambda_4^{(i)} (H^{(i)}\tilde{H}^{(i)})_{24} \\
& + \lambda_6 (R\tilde{R})_{24} + m f_1 (\Phi, M)\Big|^2 + \tfrac{1}{2}\Big| \sum_{i=1}^{n} \lambda_5^{(i)} (H^{(i)}\tilde{H}^{(i)})_1 + m f_2(\Sigma, S)\Big|^2 \\
& + \tfrac{1}{2}\sum_{i=1}^{n} \left| \lambda_4^{(i)} S H^{(i)} + \lambda_5^{(i)} \Sigma H^{(i)}\right|^2 + \tfrac{1}{2}\sum_{i=1}^{n}\left|\lambda_4^{(i)}\tilde{H}^{(i)}\Sigma + \lambda_5^{(i)}\tilde{H}^{(i)} S\right|^2 \\
& + m^2 \Big\{ |\Phi|^2 + |R|^2 + |\tilde{R}|^2 + |\Sigma|^2 + |S|^2 + \sum_i \left(|H^{(i)}|^2 + |H^{(i)}|^2\right)\Big\} \\
& + m(A-1)\{ M_1 \Phi^2 + M_2 R\tilde{R}\} + mA\{ \lambda_1 \Phi^3 + \lambda_2 \Phi R\tilde{R} + \lambda_3 \Sigma^3 \\
& + \sum_{i=1}^{n} (\lambda_4^{(i)} \Sigma + \lambda_5^{(i)} S) H^{(i)}\tilde{H}^{(i)} + \lambda_6 \Sigma R\tilde{R}\} + \frac{e^2}{2}\sum_a \Big| \sum_i y_i^\dagger T_a y_i\Big|^2
\end{aligned}
\tag{6}
$$

where the various fields in (6) denote the scalar part of the corresponding superfields. The vacuum expectation value of Φ is determined by minimizing the first term, we assume that the SU(5) symmetry is broken to SU(3)×SU(2)×U(1) by the vacuum expectation value of Φ. The vacuum expectation value of Σ is obtained by minimizing the fourth term. This gives,

$$\langle\Sigma\rangle = v\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & -\frac{3}{2} & \\ & & & & -\frac{3}{2} \end{pmatrix} \tag{7}$$

where $v \sim \sqrt{mM}$ since $f_1(\Phi, M) \sim M$. The vacuum expectation values of S, $H^{(i)}$ and $\tilde{H}^{(i)}$ are obtained by minimizing the fifth, sixth and the seventh term, and the $m^2|S|^2$ term. Assuming,

$$|\lambda_4^{(1)}/\lambda_5^{(1)}| < |2\lambda_4^{(i)}/3\lambda_5^{(i)}| \qquad \forall\, i \geq 2 \tag{8}$$

we have the following three minima, which are the possible candidates for the ground state.

(A) $\langle H^{(i)}\rangle = \langle\tilde{H}^{(i)}\rangle = 0$. $\langle S\rangle$ is obtained by minimizing,

$$|m f_2(\langle\Sigma\rangle, S)|^2 + m^2|S|^2 \tag{9}$$

If for definiteness, we take f_2 to be linear in $\langle\Sigma\rangle$ and S,

$$f_2(\langle\Sigma\rangle, S) = \beta_1 v + \beta_2 S \tag{10}$$

we get,

$$\langle S\rangle = -\beta_1\beta_2 v/(1+\beta_2^2) \tag{11}$$

and the potential at the minimum is,

$$V_{min.} = m^2\beta_1^2 v^2/(1+\beta_2^2) \tag{12}$$

(B)

$$\langle S\rangle = -v\,\lambda_4^{(1)}/\lambda_5^{(1)} \qquad \langle H_i^{(1)}\rangle = \langle \tilde{H}_i^{(1)}\rangle = 0 \qquad i=4,5$$

$$\sum_{i=1}^{3} \langle H_i^{(1)}\rangle\langle \tilde{H}_i^{(1)}\rangle = -m\, f_2\left(\langle\Sigma\rangle, -v\lambda_4^{(1)}/\lambda_5^{(1)}\right)/\lambda_5^{(1)} \tag{13}$$

$$V_{min.} = m^2 v^2 \left(\lambda_4^{(1)}/\lambda_5^{(1)}\right)^2 \tag{14}$$

(C)

$$\langle S\rangle = \tfrac{3}{2} v\,\lambda_4^{(1)}/\lambda_5^{(1)}\,, \quad \langle H_i^{(1)}\rangle = \langle \tilde{H}_i^{(1)}\rangle = 0 \qquad i=1,2,3$$

$$\sum_{i=4}^{5} \langle H_i^{(1)}\rangle\langle \tilde{H}_i^{(1)}\rangle = -m\, f_2\left(\langle\Sigma\rangle, \tfrac{3}{2} v\lambda_4^{(1)}/\lambda_5^{(1)}\right)/\lambda_5^{(1)} \tag{15}$$

$$V_{min.} = \tfrac{9}{4} m^2 v^2 \left(\lambda_4^{(1)}/\lambda_5^{(1)}\right)^2 \tag{16}$$

We want the case (C) to be the physical minimum. But as we can see, the $V_{min.}$ in this case is always larger than $V_{min.}$ in case (B). This problem may easily be avoided by adding an additional singlet field σ which couples to $R\tilde{R}$, $H\tilde{H}$ and has a self coupling proportional to σ^3, in the same way as Σ, $\langle\sigma\rangle$ contributes to the mass of H, $\tilde{H}$. Then, by suitably adjusting the parameters of the theory, we may arrange that the state of the system, where the weak doublet Higgs is massless, has less energy than the state where the color triplet higgs is massless.

However, even in the present case, if,

$$(9/4)(\lambda_4^{(1)}/\lambda_5^{(1)})^2 < \beta_1^2/(1+\beta_2^2),\quad |\beta_1\beta_2|^2/(1+\beta_2^2)^2 \tag{17}$$

and $\lambda_4^{(1)}/\lambda_5^{(1)}$ has the same sign as $-\beta_1\beta_2/(1+\beta_2^2)$, the effective potential, expressed as a function of S (by minimizing with respect to the H, $\tilde{H}$ fields) have the form of Fig.2. Thus, if we start from the S=0 state in the early universe, then, as the universe cools down, it rolls down towards the minimum given by the case (C), and gets trapped there with a very large life-time, since the seperation between the minima (B) and (C) ($\sim$ v) is much larger than the difference in energy density between the two minima ($\sim\sqrt{mv}$). In this case, the Higgs doublets acquire vev of order $\sqrt{mv}$, as given by (15). Taking $M\sim 10^{17}$GeV, $m\sim 10^3$GeV, we get $\langle S\rangle\sim\langle\Sigma\rangle\sim 10^{10}$GeV, which is the mass of the Higgs triplet, whereas $\langle H\rangle\sim\langle\tilde{H}\rangle\sim\sqrt{mv}\sim 10^{6.5}$GeV. This is 10^4 times too large compared to the physical value. A possible mechanism to push it down to the same order of magnitude as the gravitino mass will be discussed in Sec.IV.

III. INVISIBLE AXION

Besides keeping the color triplet Higgs at an intermediate mass, our model has an extra advantage that in these models one can naturally introduce an invisible axion with $f_A \sim 10^{10}$GeV. Since the presence of an invisible axion[9] requires the presence of an $SU(2)\times U(1)$ singlet field, which couples to the Higgs doublet, acquires a vev$\gtrsim 10^{9}$GeV, and still does not produce a large mass of the Higgs doublets, a natural way to introduce it is to have it as a sliding singlet. To give an example, let us introduce two singlet fields S_1 and S_2 with the coupling,

$$\alpha_1 S_1 H^{(1)} \tilde{H}^{(2)} + \alpha_2 S_2 H^{(2)} \tilde{H}^{(1)} \tag{18}$$

in the model of Sec.II. Let us, for the time being, assume that $H^{(1)}$ couples to the quark bilinear 10×10, and $\tilde{H}^{(2)}$ couples to the quark bilinear $10\times\bar{5}$. Then the model has a symmetry,

$$H^{(1)} \to e^{i\theta} H^{(1)}, \quad H^{(2)} \to e^{-i\theta} H^{(2)}, \quad \tilde{H}^{(1)} \to e^{-i\theta} \tilde{H}^{(1)}$$

$$\tilde{H}^{(2)} \to e^{i\theta} \tilde{H}^{(2)}, \quad S_1 \to e^{-2i\theta} S_1 \;, \quad S_2 \to e^{2i\theta} S_2 \tag{19}$$

together with the appropriate transformation of the quark lepton fields. This serves as the Peccei-Quinn symmetry[10]. When we minimize the potential (which now includes terms of the form $mF_{S_1} S_1^{*} f(S,S_1,S_2,\Sigma)$ etc. coming from (4)), for a certain range of values of the parameters, the minimum of the potential lies at a non-zero vev ($\sim v$) for S_1, S_2 and S, such that the mass matrix of the doublet Higgs has a zero eigenvalue, and a vev of order $\sqrt{mv}$ for the doublet Higgs. Thus the Peccei-Quinn symmetry is broken by a large vev ($\sim 10^{10}$GeV) of S_1 and S_2, giving rise to an invisible axion with $f_A \sim 10^{10}$GeV.

This particular model, however, has dimension five operators contributing to the prpton decay amplitude because of the $H^{(1)}\tilde{H}^{(2)}$ mixing term. This may be avoided by expanding the Higgs sector. We introduce new fields $H'^{(1)}$, $\tilde{H}'^{(1)}$, $H'^{(2)}$, $\tilde{H}'^{(2)}$ belonging to the 5 and $\bar{5}$ representations, a singlet S', and the coupling in the superpotential,

$$\sum_{i=1}^{2} \lambda_4'^{(i)} \Sigma \tilde{H}'^{(i)} H'^{(i)} + \sum_{i=1}^{2} \lambda_5'^{(i)} S' \tilde{H}'^{(i)} H'^{(i)} + \alpha_1' S_1 H'^{(1)} \tilde{H}'^{(2)}$$
$$+ \alpha_2' S_2 H'^{(2)} \tilde{H}'^{(1)} \qquad (20)$$

We couple $H^{(1)}$ to the quark bilinear 10×10 and $\tilde{H}'^{(2)}$ to the quark bilinear 10×$\bar{5}$. We define the Peccei-Quinn symmetry of the model as the transformations (19), together with similar transformations on the primed fields. For a finite range of values of the parameters, the vev of S, S', S_1, S_2, will be such that the mass matrix of the $H^{(i)}$, $\tilde{H}^{(i)}$ sector, as well as that of the primed sector, has a zero eigenvalue. The P-Q symmetry is then broken at a scale of order 10^{10}GeV. The primed, as well as the unprimed doublet Higgses acquire vev's of order $\sqrt{mv}$, producing the necessary masses for the quarks and leptons.

IV. A MODEL BASED ON THE SU(6) GAUGE GROUP

In this section we shall consider a model based on the SU(6) gauge group. The basic idea is the same as in Sec.II, but there are some important differences that will become clear as we proceed. The heavy fields contain an adjoint Φ, a singlet Φ_0 and $(6, \overline{6})$ pair $(R, \tilde{R})$. The light fields contain two adjoint fields χ and Σ, a singlet S_0, n pairs of $(6, \overline{6})$ of higgses $(H^{(i)}, \tilde{H}^{(i)})$ $(i=1,...n)$ and n singlets $S^{(i)}$. The superpotential is,

$$W = \lambda_1 \Phi^3 + \lambda_2 \Phi_0 \Phi^2 - M_1^2 \Phi_0 + M_2 R\tilde{R} + \lambda_3 \Phi R\tilde{R}$$
$$+ \alpha_1 S_0 \chi \Sigma + \sum_{i=1}^{n} \alpha_2^{(i)} \Sigma H^{(i)} \tilde{H}^{(i)} + \sum_{i=1}^{n} \alpha_3^{(i)} S^{(i)} H^{(i)} \tilde{H}^{(i)}$$
$$+ \beta_1 \chi R\tilde{R} + \beta_2 S_0 R\tilde{R} + \beta_3 \Sigma R\tilde{R} + \sum_{i=1}^{n} \beta_4^{(i)} S^{(i)} R\tilde{R} \qquad (21)$$

There are two important differences between this superpotential and the superpotential we considered in Secs. II and III. First, note that the sliding singlet fields $S^{(i)}$ couple to the heavy fields R, $\tilde{R}$, so that $F_{S^{(i)}}$ will now have a coupling of the form $F_{S^{(i)}} mf(\Phi, M)$ due to one loop radiative corrections. This will force $H^{(i)}\tilde{H}^{(i)}$ to acquire a vev of order $mM \sim (10^{10}\text{GeV})^2$, but, as we shall see, this is not a problem in this case, since $H_6^{(i)}$, $\tilde{H}_6^{(i)}$ may be made to acquire a vev$\sim 10^{10}$GeV. Secondly, there is no coupling of the form $SH^{(i)}\tilde{H}^{(j)}$ with $i \neq j$, so that there is no PQ symmetry which is broken by the vev of a singlet field. But this is not a problem either, since $H_6^{(i)}$'s acquire vev$\sim 10^{10}$Gev, and hence breaks any PQ symmetry which involves global transformation of the $H^{(i)}$'s.

In the supersymmetric limit, the potential obtained from the superpotential in (21) has a minimum at,

$$\langle \Phi_0 \rangle = 0 \qquad \langle \Phi \rangle = \frac{M_1}{\sqrt{6\lambda_2}} \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & -1 & & \\ & & & & -1 & \\ & & & & & -1 \end{pmatrix} \tag{22}$$

while all the other fields acquire zero vev. Again, there are other degenerate minima with different unbroken symmetry group, but we ignore them here. The potential vanishes at the minimum, and hence supersymmetry is unbroken, which can be seen by using the identity

$$\left(\langle \Phi \rangle^2\right)_{35} = \langle \Phi \rangle^2 - \frac{1}{6}\, \mathrm{Tr} \langle \Phi \rangle^2 = 0 \tag{23}$$

The vev given in (22) breaks the SU(6) symmetry to SU(3)×SU(3)×U(1) at a scale of order $M_1 \sim 10^{16}$GeV. If we now take the effect of the supersymmetry breaking terms into account, then diagrams similar to the ones shown in Fig.1 give rise to the terms in the effective action of the form,

$$m\, F_X\, f_1(\Phi, M) + m\, F_{S_0}\, f_2(\Phi, M) + m\, F_\Sigma\, f_3(\Phi, M) + m \sum_{i=1}^{n} F_{S^{(i)}}\, f_4^{(i)}(\Phi, M) \tag{24}$$

where the functions $f_1, \ldots f_4^{(i)}$ are each of order M. The F components of various light fields may now be obtained by using the equations of motion:

$$F_{\chi}^{*} = \alpha_1 (S_c \Sigma) + m f_1 (\Phi, M) + \beta_1 (R\tilde{R})_{35}$$

$$F_{S_c}^{*} = \alpha_1 (\chi \Sigma)_1 + m f_2 (\Phi, M) + \beta_2 (R\tilde{R})_1$$

$$F_{\Sigma}^{*} = \alpha_1 (S_c \chi) + \sum_{i=1}^{n} \alpha_2^{(i)} (H^{(i)} \tilde{H}^{(i)})_{35} + \beta_3 (R\tilde{R})_{35} + m f_3 (\Phi, M)$$

$$F_{S^{(i)}}^{*} = \alpha_3^{(i)} (H^{(i)} \tilde{H}^{(i)})_1 + m f_4^{(i)} (\Phi, M) + \beta_4^{(i)} (R\tilde{R})_1$$

$$F_{\tilde{H}^{(i)}}^{*} = \alpha_2^{(i)} (\Sigma H^{(i)}) + \alpha_3^{(i)} S^{(i)} H^{(i)} \tag{25}$$

Using the fact that $\langle R \rangle, \langle \tilde{R} \rangle = 0$ (since they have mass of order M), and that f_1 and f_3 are proportional to $\langle \Phi \rangle$, we may minimize the potential by setting each of the F's in (25) to be zero by vev of various fields of the form:

$$\langle \Sigma \rangle = a_1 v \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & -1 & & \\ & & & & -1 & \\ & & & & & -1 \end{pmatrix} \qquad \langle \chi \rangle = v \begin{pmatrix} a_2 & & & & & \\ & a_2 & & & & \\ & & a_2 & & & \\ & & & a_3 & & \\ & & & & a_3 & \\ & & & & & -3a_2 - 2a_3 \end{pmatrix}$$

$$\langle S_c \rangle = a_4 v$$

$$\langle H_6^{(i)} \rangle = \langle \tilde{H}_6^{(i)} \rangle = a_5^{(i)} v \qquad 1 \le i \le n$$

$$\langle H_m^{(i)} \rangle = \langle \tilde{H}_m^{(i)} \rangle = 0 \qquad 1 \le i \le n, \quad 1 \le m \le 5$$

$$\langle S^{(i)} \rangle = (\alpha_2^{(i)} / \alpha_3^{(i)})\, a_1 v \tag{26}$$

where $a_1,\ldots a_5^{(i)}$ are numbers of order unity, calculable in terms of the functions $f_1,\ldots f_4^{(i)}$ and $v=\sqrt{mM}\sim 10^{10}$GeV. The important point to note is that the vev of the sliding singlets $S^{(i)}$, which keep $H_6^{(i)}$ and $\tilde{H}_6^{(i)}$ massless, also keeps $H_4^{(i)}$, $H_5^{(i)}$, $\tilde{H}_4^{(i)}$ and $\tilde{H}_5^{(i)}$ massless. One linear combination of these fields is absorbed by the gauge bosons which then become massive, thus breaking the SU(3)×SU(3)×U(1) symmetry to SU(3)×SU(2)×U(1). [At this point we should mention that there are other degenerate vacua, where the gauge group is broken to SU(3)×U(1), or to SU(3) at a scale of order 10^{10}GeV, since the vev of the different H fields may be directed in different directions. A detailed study of the contribution from the higher order terms is needed to determine which vacuum has the lowest energy. In the rest of the paper we shall assume that the SU(3)×SU(2)×U(1) phase has the lowest energy.] Another linear combination of the fields gets mass from the D terms, so as to produce a complete massive vector supermultiplet. The rest of the $H_m^{(i)}$, $\tilde{H}_m^{(i)}$ (m=4,5) fields remain massless at this level. These fields may then acquire vev of order m through radiative corrections as discussed in Ref.[11].

The quark fields may be choosen to belong to the $\bar{6}$, $\bar{6}$, 15 and 20 representations of SU(6), which we denote by $Q_{\bar{6}}^{(1)}$, $Q_{\bar{6}}^{(2)}$, Q_{15}, and Q_{20} respectively. They get mass from the following coupling to the Higgs fields,

$$\sum_i \{ \gamma_1^{(i)} Q_{\bar{6}}^{(1)} Q_{15} \tilde{H}^{(i)} + \gamma_2^{(i)} Q_{\bar{6}}^{(2)} Q_{15} \tilde{H}^{(i)} + \gamma_3^{(i)} Q_{15} Q_{20} H^{(i)} \}$$

$$+ \gamma_4 Q_{20} Q_{20} \Sigma \tag{27}$$

To see that this produces the correct low energy spectrum of the quark lepton fields, let us consider the decomposition of these fields in terms of representations of the SU(5) subgroup of SU(6):

$$\bar{6} \to \bar{5} + 1$$

$$15 \to 10 + 5$$

$$20 \to 10 + \overline{10} \qquad (28)$$

Thus the first term produces a mass term:

$$\left\{ \left(\sum_i \gamma_1^{(i)} \langle \tilde{H}_6^{(i)} \rangle \right) Q^{(1)}_{\bar{5}(\bar{6})} + \sum_i \gamma_2^{(i)} \langle \tilde{H}_6^{(i)} \rangle \, Q^{(2)}_{\bar{5}(\bar{6})} \right\} Q_{5(15)} \qquad (29)$$

where $Q_{5(15)}$ denotes the part of Q_{15} which transforms as the 5 of SU(5), and similarly for the others. (29) produces a mass of order v for the 5 component of Q_{15} and a linear combination of the $\bar{5}$ components of $Q_{\bar{6}}^{(1)}$ and $Q_{\bar{6}}^{(2)}$. The orthogonal linear combination does not acquire a mass at this level, but combines with the 10 component of Q_{15} to get a mass of order m from the third term in (27). This arises due to the vev of $\tilde{H}_5^{(i)}$ of order m.

The third and the fourth terms in (27), on the other hand, produces a mass term of the form:

$$\left(\sum_{i=1}^{n} \gamma_3^{(i)} \langle H_6^{(i)} \rangle \, Q_{10(15)} + \gamma_4 \langle \Sigma \rangle \, Q_{10(20)} \right) Q_{\overline{10}(20)} \qquad (30)$$

Thus a particular linear combination of $Q_{10(15)}$ and $Q_{10(20)}$ combines with $Q_{10(20)}$ to acquire a mass of order v. The orthogonal combination of $Q_{10(15)}$ and $Q_{10(20)}$ remains massless at this order, but gets a mass of order m through the combination,

$$\sum_{i=1}^{n} \gamma_3^{(i)} \langle H_5^{(i)} \rangle \, Q_{10(15)} \, Q_{10(20)} \tag{31}$$

It is easy to introduce a Peccei-Quinn symmetry in this model and to prevent dimension five operators to contribute to the proton decay amplitude, by setting some of the γ's to be zero. (e.g. all the γ's except $\gamma_1^{(1)}$, $\gamma_2^{(2)}$, $\gamma_3^{(3)}$ and γ_4 are zero.) The Peccei-Quinn symmetry, involving unequal phase transformations on the $H^{(i)}$'s, is broken at a scale of order $v \sim 10^{10}$GeV due to the vev of $H_6^{(i)}$, thus giving rise to an invisible axion with decay constant of order 10^{10}GeV.

In this class of models, the SU(6) symmetry is spontaneously broken down to SU(3)×SU(3)×U(1) at a scale of order 10^{16}GeV, which breaks down to SU(3)×SU(2)×U(1) at a scale of order 10^{10}GeV. This may be a good news for the prediction of $\sin^2\theta_w$, since it usually comes out to be too high in supersymmetric grand unified theories. In the present model, with $\sin^2\theta_w = {\cdot}215$, and three pairs of light Higgs doublets, the $SU(2)^w$ and $SU(3)^c$ coupling constants meet at a scale of order 10^{10}GeV. Above this scale the coupling constants of the two unbroken SU(3) subgroups run together and meet the coupling constant of the U(1) subgroup at a scale of order 10^{16}GeV.

CONCLUSION

In this paper, we have given examples of spontaneously broken, locally supersymmetric grand unified theories, with a natural solution of the hierarchy problem. In these models, the doublet Higgs, whose vev breaks the $SU(2)\times U(1)$ symmetry, and its color triplet partner are massless in the supersymmetric limit. Due to supersymmetry breaking, and the existence of light $SU(3)\times SU(2)\times U(1)$ singlet fields, the color triplet Higgs acquires a mass of order $v=\sqrt{mM}$ from the one loop radiative corrections, where M is the grand unification scale and m is the gravitino mass. The doublet Higgs is prevented to have a mass of order v by the sliding singlet mechanism[4]. But in the simplest SU(5) grand unified theory, it acquires a vev of order $\sqrt{mv}$. Taking $m\sim 10^3$GeV and $M\sim 10^{17}$GeV, we get a colored Higgs triplet of mass of order 10^{10}GeV, and a Higgs doublet of vev of order $10^{6.5}$GeV. However, by starting with a bigger gauge group, e.g. SU(6), we may construct models where the $SU(2)^w\times U(1)$ breaking scale is of order m. In these models, we necessarily have an unbroken gauge group bigger than $SU(3)\times SU(2)\times U(1)$ above 10^{10}GeV, which breaks down to $SU(3)\times SU(2)\times U(1)$ at this scale.

In the models of the kind considered in this paper, one can naturally introduce invisible axions without the need of fine tuning any parameter. The axion decay constant turns out to be of order $v \sim 10^{10}$GeV, which lies in the narrow range of values allowed by the present cosmology[12].

There are however many technical details which remain to be studied (e,g. the evolution of various coupling constants and mass parameters, as governed by the renormalization group equations, the domain wall problem, etc.). The possibility of constructing a realistic model, based on the ideas developed in this paper, is under investigation.

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FIGURE CAPTIONS

FIG.1: Typical supergraphs contributing to $f_1(\Phi,M)$ ((a) and (b)) and $f_2(\Sigma,S)$ ((c) and (d)) respectively. The external lines, marked F, imply that we choose the F components of the superfields from these lines. From all other external lines, we choose the first components of the superfields.

FIG.2: The effective potential as a function of S.

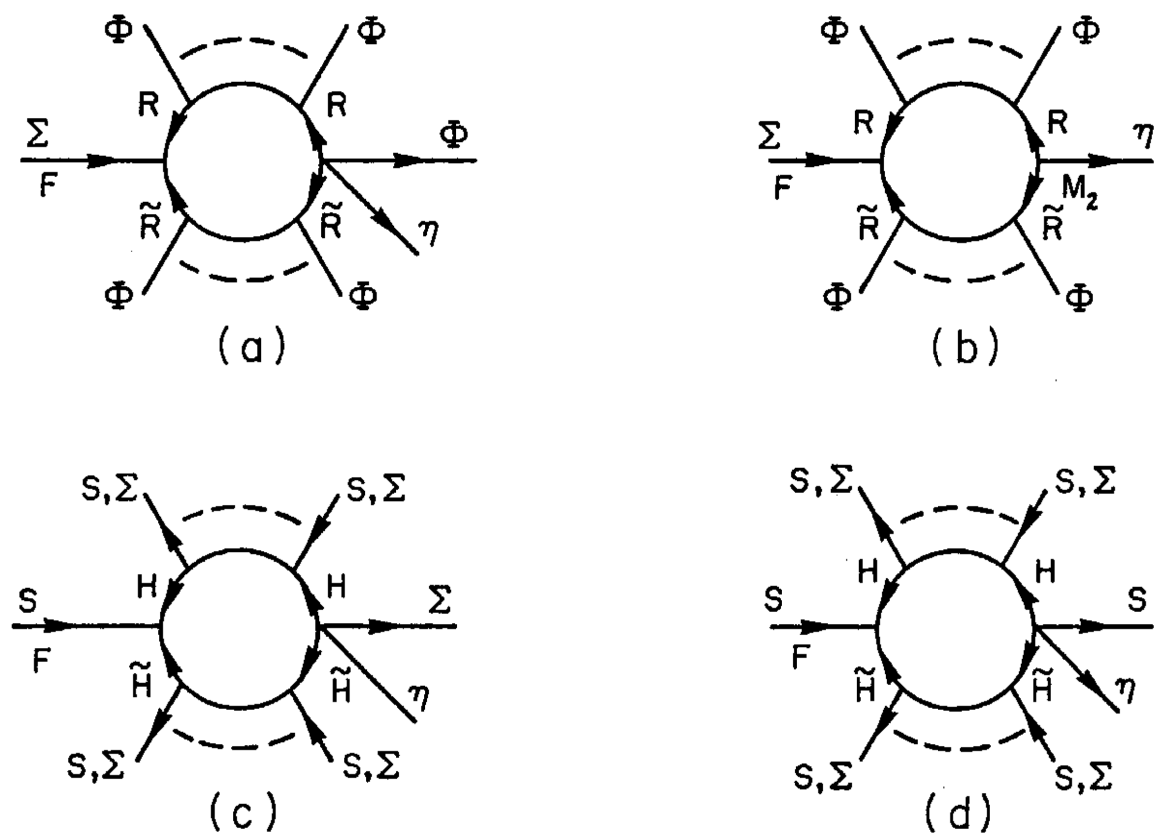

FIG. 1

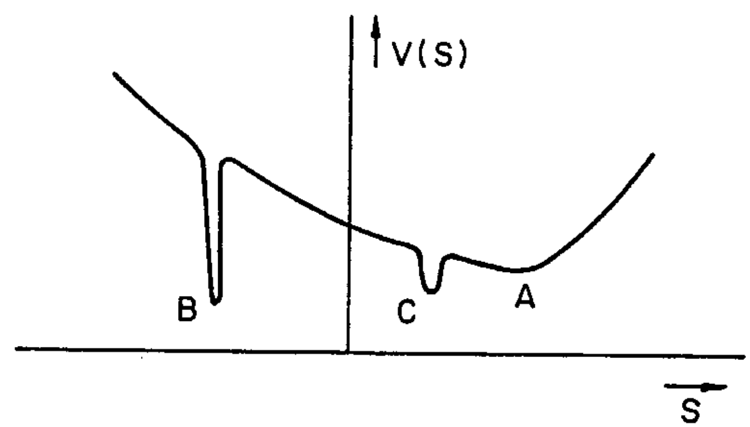

FIG. 2